On Using Model Checking for the Certification of Iterated Belief Changes

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Agenda

- The Certification Problem
- Encoding as Model-Checking Problem
- Webtool Alchourron
- Evaluation and Improvements

The Certification Problem

(Iterated) Belief Change

- Agents have to adapt their beliefs according to potentially conflicting information
- Iterated belief change: Modeled by operators over epistemic states[1].
- Often propositional language $\mathcal{L}$ over finite signature Σ
- In the following: $\alpha, \beta \in \mathcal{L}$ denote sentences, Ω denotes set of interpretations

[1] In contrast, classical belief revision uses belief sets or belief bases as states.

Belief Revision on Epistemic States [1]

- Epistemic state: abstract entity $\Psi \in \mathcal{E}$, equipped with a deductively closed set of currently held beliefs $\mathrm{Bel}(\Psi)$
- belief change operator $\circ : \mathcal{E} \times \mathcal{L} \to \mathcal{E}$
- we assume syntax-independence for $\circ$:
  - $\text{if } \alpha \equiv \beta, \text{ then } \Psi \circ \alpha = \Psi \circ \beta$
- instantiation of $\mathcal{E}$ here: total preorders over Ω that fulfil the faithfulness condition $\mathrm{Mod}(\mathrm{Bel}(\Psi)) = \min(\Omega, \leq)$
- therefore every $\leq \in \mathcal{E}$ entirely describes an epistemic state

Postulates

- postulates place restrictions on individual changes or whole operators
- operators are classified depending on what postulates they fulfil
- semantic and syntactic postulates
- for example Darwiche-Pearl postulates for revision [1], here CR1
  - if $\omega_1, \omega_2 \in \mathrm{Mod}(\alpha)$, then $\omega_1 \leq_\Psi \omega_2 \Leftrightarrow \omega_1 \leq_{\Psi \circ \alpha} \omega_2$

Certification Problem

CERTIFICATION-PROBLEM
Given: A belief change operator $\circ$ and a postulate P
Question: Does $\circ$ satisfy the postulate P?

- A singular belief change from Ψ to Ψ' by α, i.e.: Does $\Psi \circ \alpha = \Psi'$ hold?
- A sequence of belief changes $\Psi_1 \circ \alpha_1 = \Psi_2$, and $\Psi_2 \circ \alpha_2 = \Psi_3$, and ...
- All singular belief changes on a state Ψ, i.e. the set $\{(\Psi_1, \alpha, \Psi_2) \in \circ \mid \Psi = \Psi_1\}$

Encoding as Model-Checking Problem

Approach

- Define a first-order fragment FO^{TPC} to encode change in epistemic states with new information
- Build a FO^{TPC}-structure A_C for a concrete belief change $C = (\Psi, \alpha, \Psi')$
- Load postulate as formula φ and evaluate $\mathcal{A}_C \models \varphi$

Language for Postulates

Predicate	Intended meaning	Exemplary appearance
$Mod(w, x)$	w is a model of x	$\omega \in \mathrm{Mod}(\Psi), \omega \in \mathrm{Mod}(\alpha)$
$LessEQ(w_1, w_2, e)$	$w_1 \leq w_2$ in e	$\omega_1 \leq_\Psi \omega_2$
$Int(w)$	w is an interpretation	$\omega \in \Omega$
$ES(e)$	e is an epistemic state	$\Psi \in \mathcal{E}$
$Form(a)$	a is a formula	$\alpha \in \mathcal{L}$
Function	**Intended meaning**	**Exemplary appearance**
$op(e_0, a)$	$op(e_0, a)$ is a result of changing e_0 by a	$\Psi \circ \alpha = \Psi'$
$or(a, b)$	propositional disjunction	$\mathrm{Bel}(\Psi \circ (\alpha \vee \beta)) = \ldots$
$not(a)$	propositional negation	$\neg\alpha \notin \mathrm{Bel}(\Psi \circ \alpha)$

$$LogImpl(x, y) := \forall w. Int(w) \rightarrow (Mod(w, x) \rightarrow Mod(w, y))$$

Structure $\mathcal{A}_C$

Universe	$U^{\mathcal{A}_C} = \Omega \cup \{\Psi_0, \Psi_1\} \cup \mathcal{P}(\Omega)$
Predicates	
$Mod^{\mathcal{A}_C}$	$= \{(\omega, x) \mid x \in \mathcal{P}(\Omega) \cup \{\Psi_0, \Psi_1\}, \omega \in \mathrm{Mod}(x)\}$
$Int^{\mathcal{A}_C}$	$= \Omega$
$ES^{\mathcal{A}_C}$	$= \{\Psi_0, \Psi_1\}$
$Form^{\mathcal{A}_C}$	$= \mathcal{P}(\Omega)$
$LessEQ^{\mathcal{A}_C}$	$= \{(\omega_1, \omega_2, \Psi_i) \mid \omega_1 \leq_{\Psi_i} \omega_2\}$
Functions	
$or^{\mathcal{A}_C}$	$= \lambda\alpha_1, \alpha_2.\, \alpha_1 \cup \alpha_2$ $\qquad e_0^{\mathcal{A}_C} = \Psi_0$
$not^{\mathcal{A}_C}$	$= \lambda\alpha_1.\, \Omega \setminus \alpha_1$ $\qquad a^{\mathcal{A}_C} = \mathrm{Mod}(\alpha)$
$op^{\mathcal{A}_C}$	$= (\{(\Psi, \beta, \Psi) \mid \beta \in \mathcal{P}(\Omega), \Psi \in \{\Psi_0, \Psi_1\}\} \setminus \{(\Psi_0, \alpha, \Psi_0\}) \cup \{(\Psi_0, \alpha, \Psi_1)\}$

Webtool Alchourron

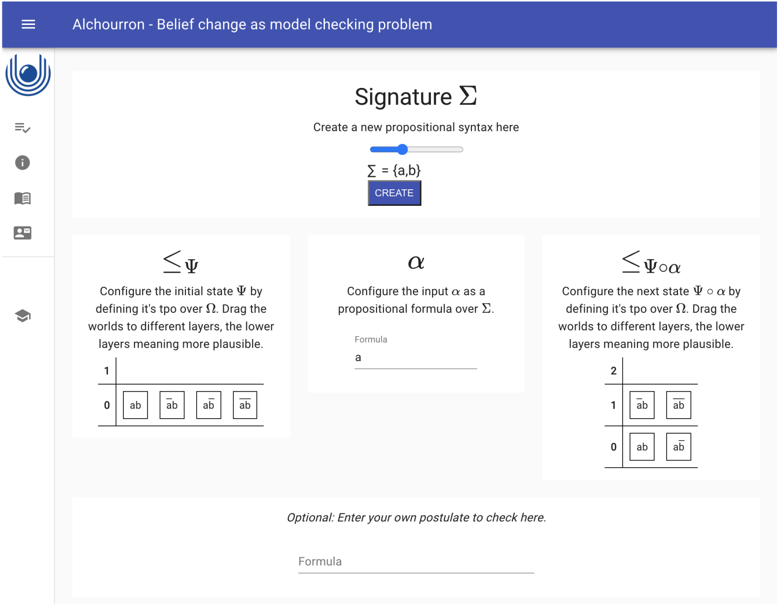
Implementation

- Available online at `https://www.fernuni-hagen.de/wbs/alchourron/`
- Client-Server architecture
- Backend: Own Java library, Frontend: Browser with web components
- Loads postulates in TPTP syntax [3] using scala-tptp-parser [2]

Postulates in TPTP Syntax

```
fof(
    'CR1',
    postulate,
    ! [W1,W2] : (
        (int(W1) & int(W2) & mod(W1, A) & mod(W2, A))
        =>
        (lesseq(W1, W2, E0) <=> lesseq(W1, W2, op(E0, A)))
    )
).
```

Input



Alchourron - Belief change as model checking problem
Signature Σ
Create a new propositional syntax here
Σ = {a,b}
CREATE
$\leq_\Psi$
Configure the initial state Ψ by defining it's tpo over Ω. Drag the worlds to different layers, the lower layers meaning more plausible.
1
0 ab $\bar{a}b$ $a\bar{b}$ $\bar{a}\bar{b}$
α
Configure the input α as a propositional formula over Σ.
Formula
a
$\leq_{\Psi\circ\alpha}$
Configure the next state $\Psi \circ \alpha$ by defining it's tpo over Ω. Drag the worlds to different layers, the lower layers meaning more plausible.
2
1 $\bar{a}b$ $\bar{a}\bar{b}$
0 ab $a\bar{b}$
Optional: Enter your own postulate to check here.
Formula

Output

$$C = (\Psi, \alpha, \Psi \circ \alpha)$$

Finally, click "Check Postulates" to verify which postulates are satisfied by the entered belief change.

CHECK POSTULATES

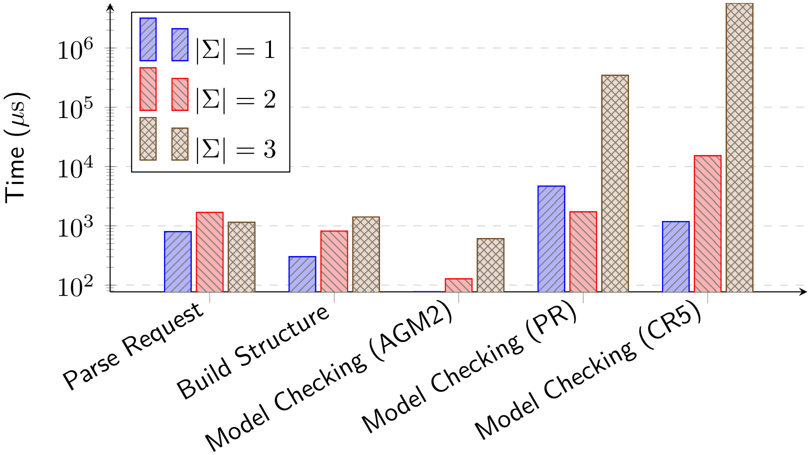
Name	Formula	Satisfied?
Success	FORALL W1. (NOT (Int(W1) AND Mod(W1, op[E0, A])) OR Mod(W1, A))	✓
Vacuity	FORALL W1. (NOT (Int(W1) AND (Mod(W1, E0) AND Mod(W1, A))) OR Mod(W1, op[E0, A]))	✓

Evaluation and Improvements

Performance Questions

- assumption: size of signature has biggest impact on performance
- possible bottlenecks: parsing request, building A_C, model-checking postulate
- method: measure average times for belief change that fulfils all postulates

Measurement Results



- biggest factor: number of quantifiers in postulate formula

Work done

- better parallelism for postulate evaluation and quantified formula evaluation
- response time from 12.5s to 3.9s for signature of size three

Pros and Cons

- \+ Easy to extend with new postulates
- \+ Completely automated
- \+ Potentially able to provide counter examples
- − Performance

Future work

- Extend approach to more sub-problems (i.e. whole operators)
- Performance: Improve formula evaluation

Thank you

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- Try yourself online at
  `https://www.fernuni-hagen.de/wbs/alchourron/`

References

[1] Adnan Darwiche and Judea Pearl. "On the logic of iterated belief revision". In: *Artificial Intelligence* 89.1-2 (Jan. 1997), pp. 1–29. DOI: 10.1016/s0004-3702(96)00038-0. URL: https://doi.org/10.1016/s0004-3702(96)00038-0.

[2] Alexander Steen. *scala-tptp-parser*. Version v1.3. The TPTP syntax specification may be accessed at http://tptp.org/TPTP/SyntaxBNF.html. Apr. 2021. DOI: 10.5281/zenodo.4672395. URL: https://doi.org/10.5281/zenodo.4672395.

[3] G. Sutcliffe. "The TPTP Problem Library and Associated Infrastructure. From CNF to TH0, TPTP v6.4.0". In: *Journal of Automated Reasoning* 59.4 (2017), pp. 483–502.