Implementing a Structured Approach to Belief Revision by Deterministic Switching Between Total Preorders

Masters Colloquium

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Agenda

Motivation

Deterministic Multiform Revision

Implementation

Evaluation and Improvements

Motivation

Motivation

- Agents have to adapt their beliefs according to potentially conflicting information
- Operators, classified axiomatically by syntactic/semantic postulates
- Postulates need to be certified for new operators
- Can classification be automated?

Deterministic Multiform Revision

Extended Epistemic States

- AGM Belief Change [AGM85]
- Belief Revision on Epistemic States [DP97]
- Extended epistemic states (e.g. [BM11])

Uniform Revision [Ara20]

- One, static tpo $\preceq$
- Family of tpos due to URF1: For any $\omega_1, \omega_2 \notin \mathrm{Mod}(\Psi)$, $\omega_1 \leq_\Psi \omega_2$ iff $\omega_1 \preceq \omega_2$
- Select a tpo $\leq_B$ for any belief set B by faithfulness to B
- AGM revision operator $B * \alpha = th(min(\mathrm{Mod}(\alpha), \leq_B))$ [KM91]

Deterministic Multiform Systems

- Extends uniform revision (different contexts, history...)
- Multiform system $M = (T, E)$
- Deterministic and syntax-independent

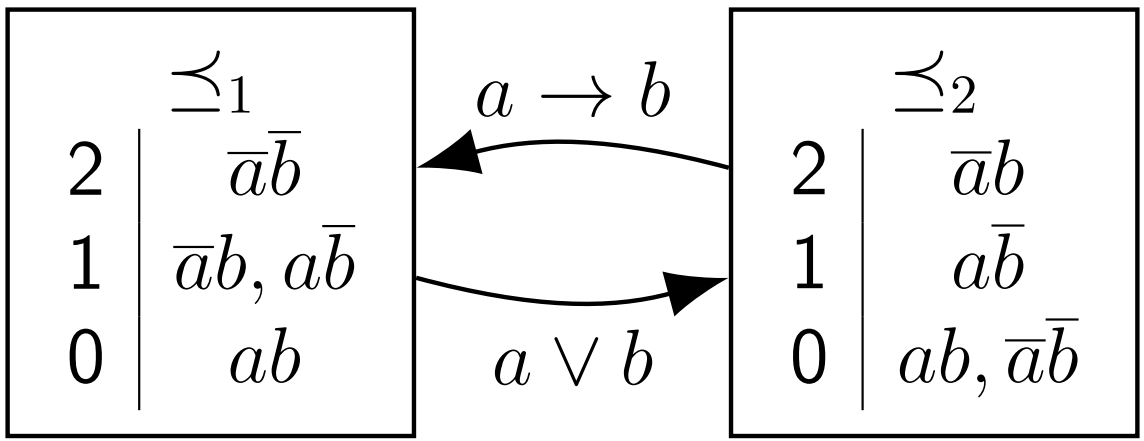


Figure: Example of a multiform system as a graph

Deterministic Multiform System States

- Tupel $s^M = (\mathbb{B}, \preceq)$
- DMF system state assignment $\Psi \mapsto s^M_\Psi$ iff $Bel(\Psi) = \mathbb{B}_\Psi$
- Operator for belief set revision (UR operator for $\preceq_\Psi$): $*_{s^M_\Psi} : \mathcal{B} \times \mathcal{L} \to \mathcal{B}$

Deterministic Multiform System Revision

- Revision operator $\circ : \mathcal{E} \times \mathcal{L} \to \mathcal{E}$
- Called DMF revision operator if there is a DMF system state assignment such that $s^M_{\Psi\circ\alpha} = (\mathbb{B}_{\Psi\circ\alpha}, \preceq_{\Psi\circ\alpha})$:

$$\mathbb{B}_{\Psi\circ\alpha} = \mathbb{B}_\Psi *_{s^M_\Psi} \alpha$$

$$\preceq_{\Psi\circ\alpha} = \begin{cases} \preceq' & \text{if } (\preceq_\Psi, \alpha, \preceq') \in E \\ \preceq_\Psi & \text{otherwise} \end{cases}$$

Derived-faithful tpos

Definition (Derived-faithful tpo to a DMF system state)

A tpo $\leq_{s_\Psi^M}$ is called derived-faithful to a *DMF system state* $s_\Psi^M = (\mathbb{B}_\Psi, \preceq_\Psi)$ iff:

1. It is *faithful* to the belief set $\mathbb{B}_\Psi$ of s_Ψ^M: $\mathbb{B}_\Psi = th(min(\mathrm{Mod}(\top), \leq_{s_\Psi^M}))$
2. Other conditional beliefs encoded by it align with the state context $\preceq_\Psi$: For any $\omega_1, \omega_2 \notin \mathrm{Mod}(\mathbb{B}_\Psi)$, $\omega_1 \leq_{s_\Psi^M} \omega_2$ iff $\omega_1 \preceq_\Psi \omega_2$

Compute derived-faithful tpo

Input: DMF System State $s_\Psi^M = (\mathbb{B}_\Psi, \preceq_\Psi)$
Result: Derived-faithful tpo $\leq_{s_\Psi^M}$
$\leq_{s_\Psi^M} = \emptyset$
foreach $(\omega, \omega') \in \preceq_\Psi$ **do**
  if $\omega, \omega' \notin \mathrm{Mod}(\mathbb{B}_\Psi)$ **then**
    $\leq_{s_\Psi^M} := \leq_{s_\Psi^M} \cup \{(\omega, \omega')\}$
  end
end
foreach $\omega \in \mathrm{Mod}(\mathbb{B}_\Psi)$ **do**
  $\leq_{s_\Psi^M} := \leq_{s_\Psi^M} \cup \{(\omega, \omega') \mid \omega' \in \Omega - \mathrm{Mod}(\mathbb{B}_\Psi)\}$
end

DMF Revision Operator

1. Construct $\leq_{s_\Psi^M}$ for the initial DMF system state s_Ψ^M, encoding a revision operator $*_{s_\Psi^M}$
2. Compute $\mathbb{B}_{\Psi \circ \alpha} = \mathbb{B}_\Psi *_{s_\Psi^M} \alpha$
   - 2.1 Minimal models of α in $\leq_{s_\Psi^M}$ define the belief set of $\Psi \circ \alpha$: $\mathrm{Mod}(Bel(\Psi \circ \alpha)) = min(\mathrm{Mod}(\alpha), \leq_{s_\Psi^M})$
   - 2.2 Set $\mathbb{B}_{\Psi \circ \alpha} := th(min(\mathrm{Mod}(\alpha), \leq_{s_\Psi^M}))$ (minimize using Quine–McCluskey [McC56])
3. Find $\preceq_{\Psi \circ \alpha}$
   - 3.1 Outgoing edge e from context tpo $\preceq_\Psi$: $(\preceq_\Psi, \beta, \preceq')$ with $\beta \equiv \alpha$
   - 3.2 If e exists, set $\preceq_{\Psi \circ \alpha} := \preceq'$, otherwise $\preceq_{\Psi \circ \alpha} := \preceq_\Psi$
4. Set $s_{\Psi \circ \alpha}^M := (\mathbb{B}_{\Psi \circ \alpha}, \preceq_{\Psi \circ \alpha})$

Implementation

Certification Problem [SH21]

- Given: A belief change operator $\circ$ and a postulate P
- Question: Does $\circ$ satisfy the postulate P?
- Sub-Problems: Singular belief change, consecutive belief changes, all belief changes from one state...

Encoding as Model-Checking Problem

- Define a first-order fragment FO^{TPC} to encode change in epistemic states with new information
- Build a FO^{TPC}-structure A_C for a concrete belief change $C = (\Psi, \alpha, \Psi')$
- Load postulate as formula φ and evaluate $\mathcal{A}_C \models \varphi$

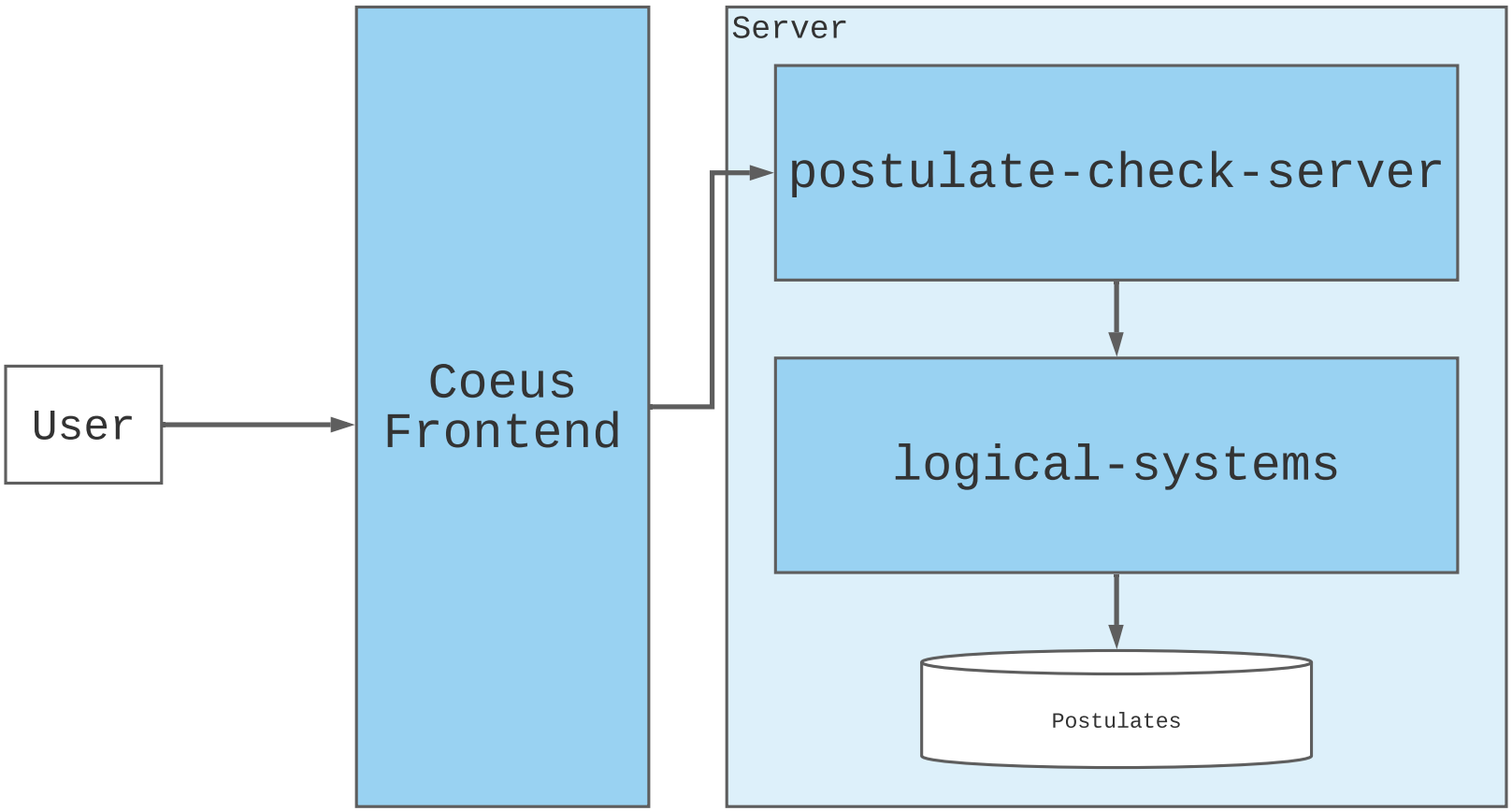
FAIR research software [Has+20]

- Findable: Be easy to find and cite
- Accessible: Provide a way to download a software snapshot
- Interoperable: Use existing standards and target as many runtime platforms as possible
- Reusable: Modular and easy to extend, follow good software development practices

Implementation Overview

- Open Source (GitHub)
- Citation and Snapshots with Zenodo [EO13]
- Frontend: TypeScript, Web Components/React
- Backend: Java
- Automated CI/CD (GitHub Actions)
- Deployment with Docker

Software Architecture



Postulates in TPTP Syntax

```
fof(
    'CR1',
    postulate,
    ![W1,W2] : (
        (int(W1) & int(W2) & mod(W1, A) & mod(W2, A))
        =>
        (lesseq(W1, W2, E0) <=> lesseq(W1, W2, op(E0, A)))
    )
).
```

Textual Representation: TPOs

- Encode a world $\omega = a\bar{b}$
- As variables that are interpreted as true: ["a"]
- By a binary number, alphabetically ordering variables, setting true ones to 1: $\omega = a\bar{b} = 10$

Textual Representation: TPOs

2	$ab,\ \overline{a}\overline{b}$
1	
0	$a\overline{b}, \overline{a}b$

Figure: Ordinal conditional function on $\{ab, a\overline{b}, \overline{a}b, \overline{a}\overline{b}\}$

Textual Representation: TPOs

Listing: Ocf represented as JSON

```
{
        "signature": ["a", "b"],
        "ranks": [
                [
                        ["a"],
                        ["b"]
                ],
                [],
                [
                        ["a", "b"],
                        []
                ]
                ]
}
```

Textual Representation: TPOs

Listing: Ocf in worldlist binary

```
00000001  // Version: 1
00000000000000000000000000000010
// Signature size: 2

00000000000000000000000000000010
// Rank size: 2
00000000000000000000000000000001
// Gap size: 1

00000010 // World 2
00000001  // World 1

00000000000000000000000000000010
// Rank size: 2
00000000000000000000000000000000
// Gap size: 0

00000011 // World 3
00000000 // World 0
```

Textual Representation: TPOs

Listing: Ocf in ranklist binary

```
00000001 // Version: 1
00000000000000000000000000000010
// Signature size: 2

00000000000000000000000000000010
// World 0 is on rank 2
00000000000000000000000000000000
// World 1 is on rank 0
00000000000000000000000000000000
// World 2 is on rank 0
00000000000000000000000000000010
// World 3 is on rank 2
```

Textual Representation: TPOs

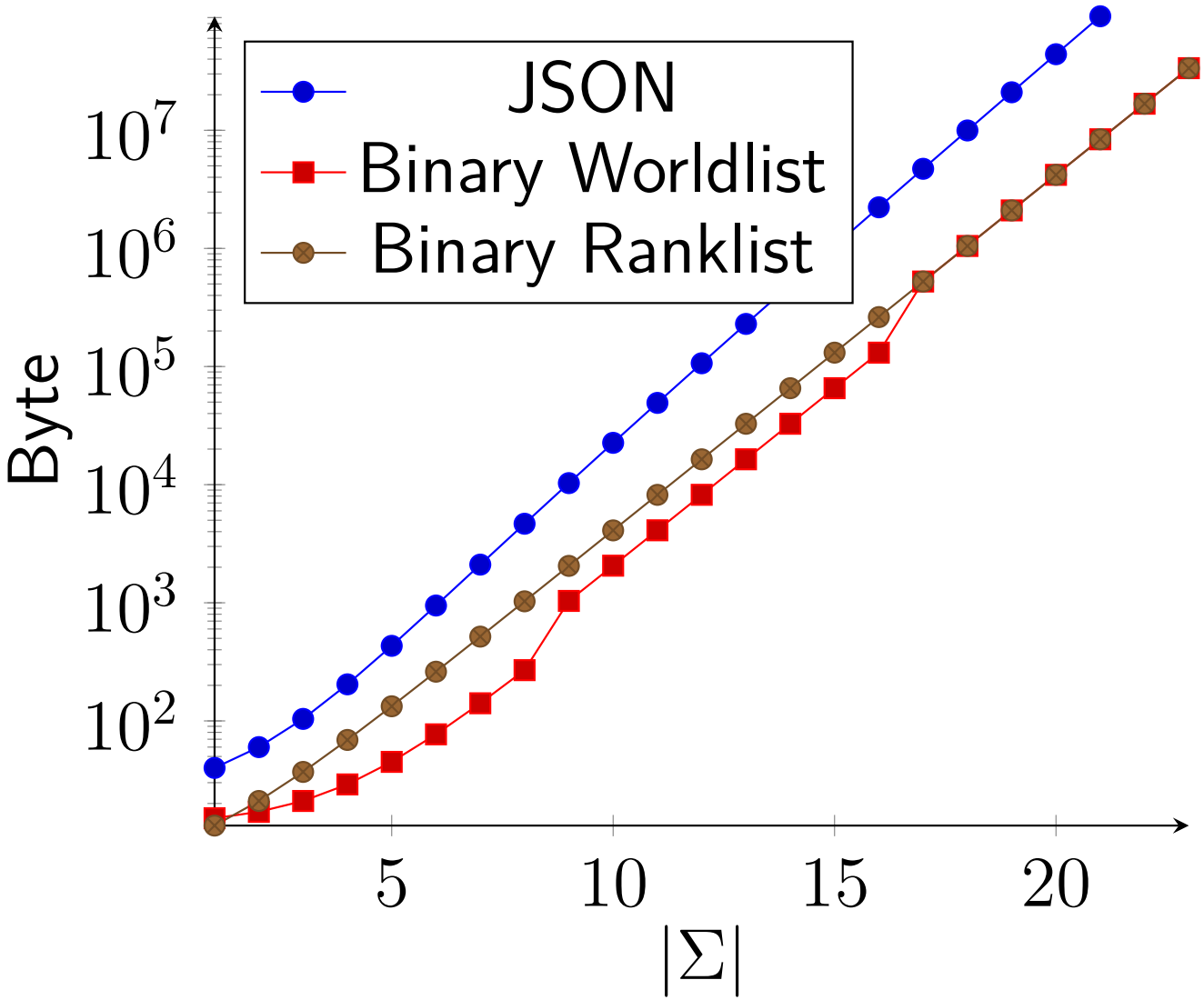
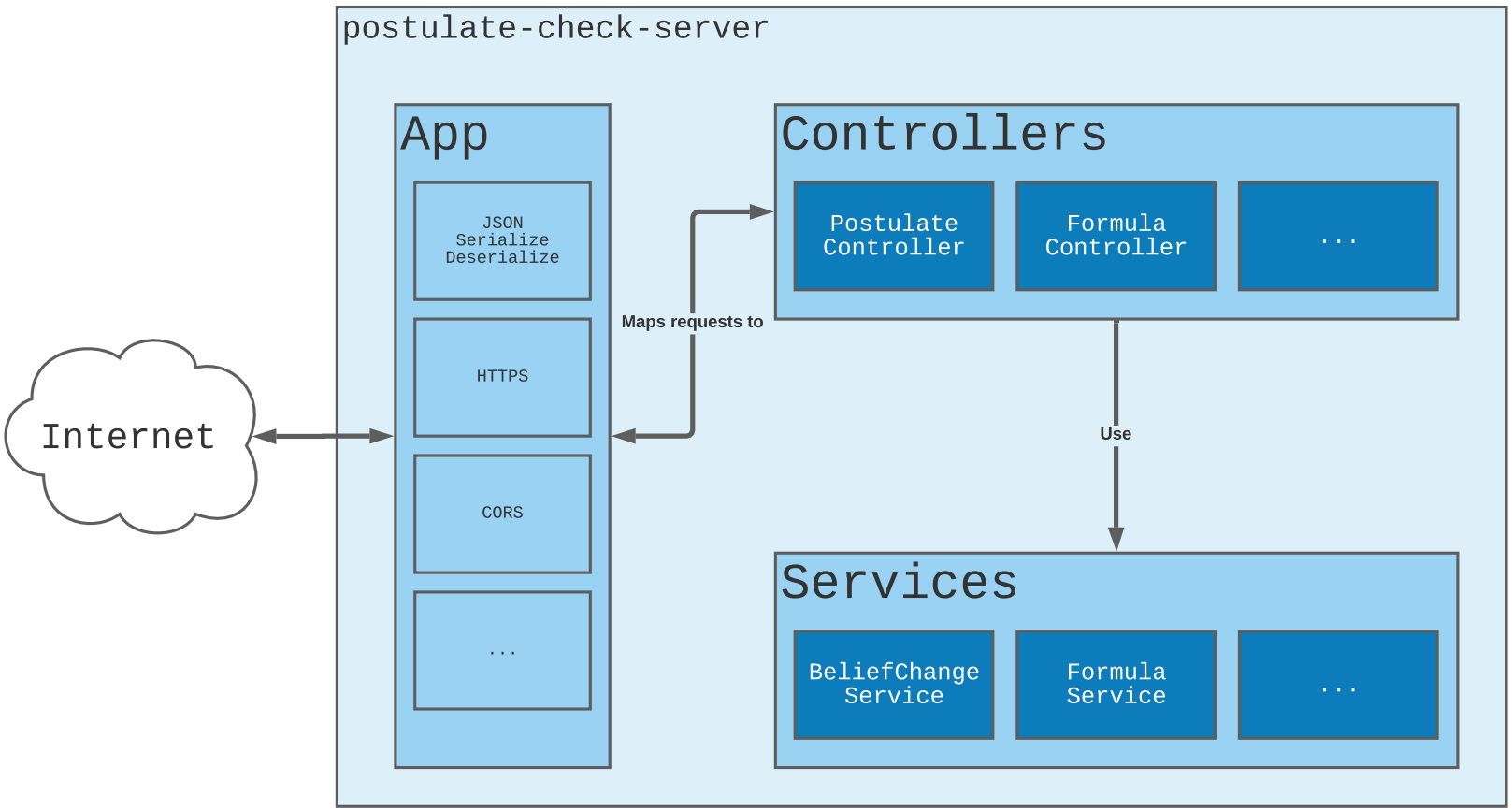
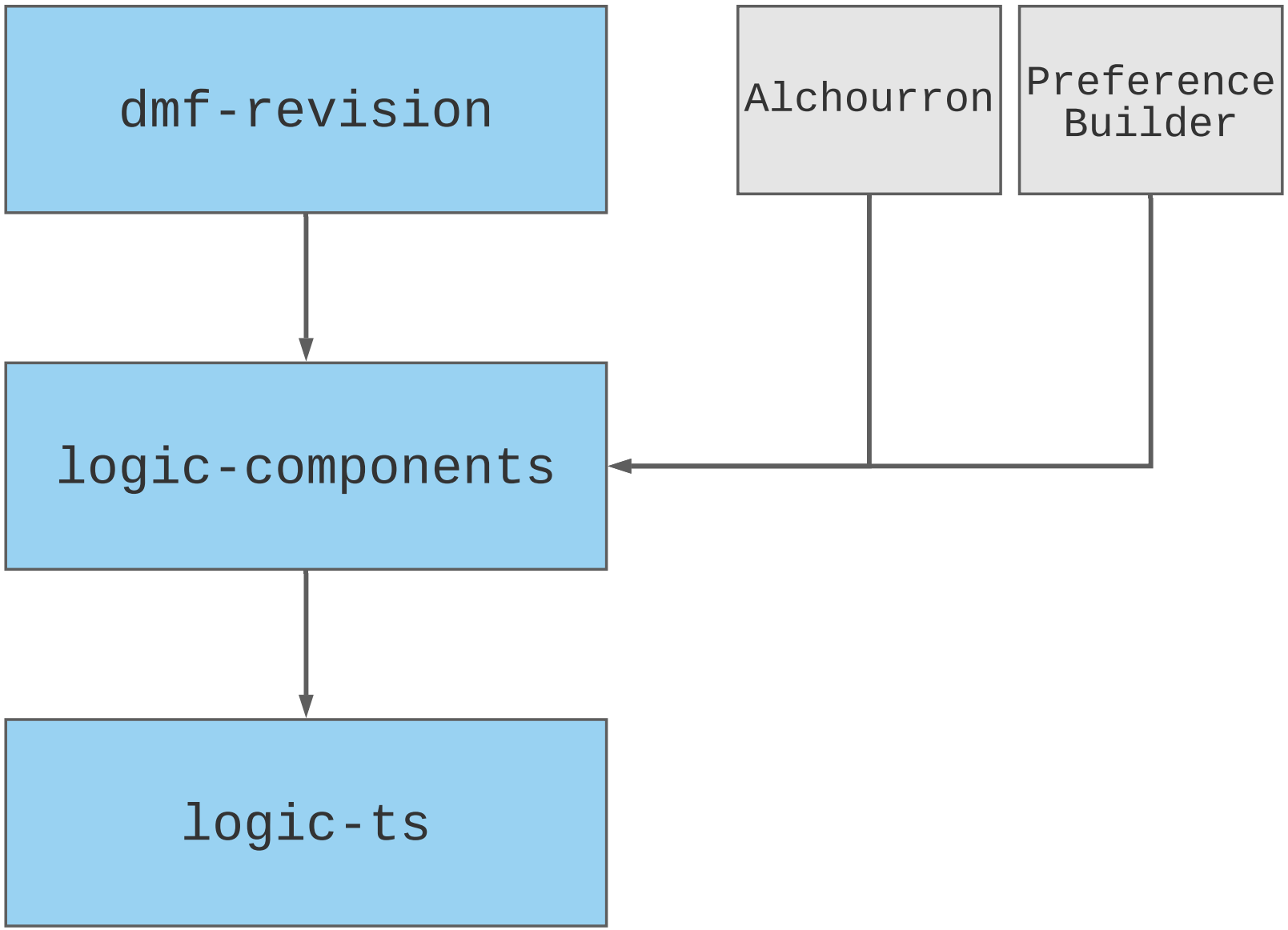


Figure: Size of serialized tpo (log)

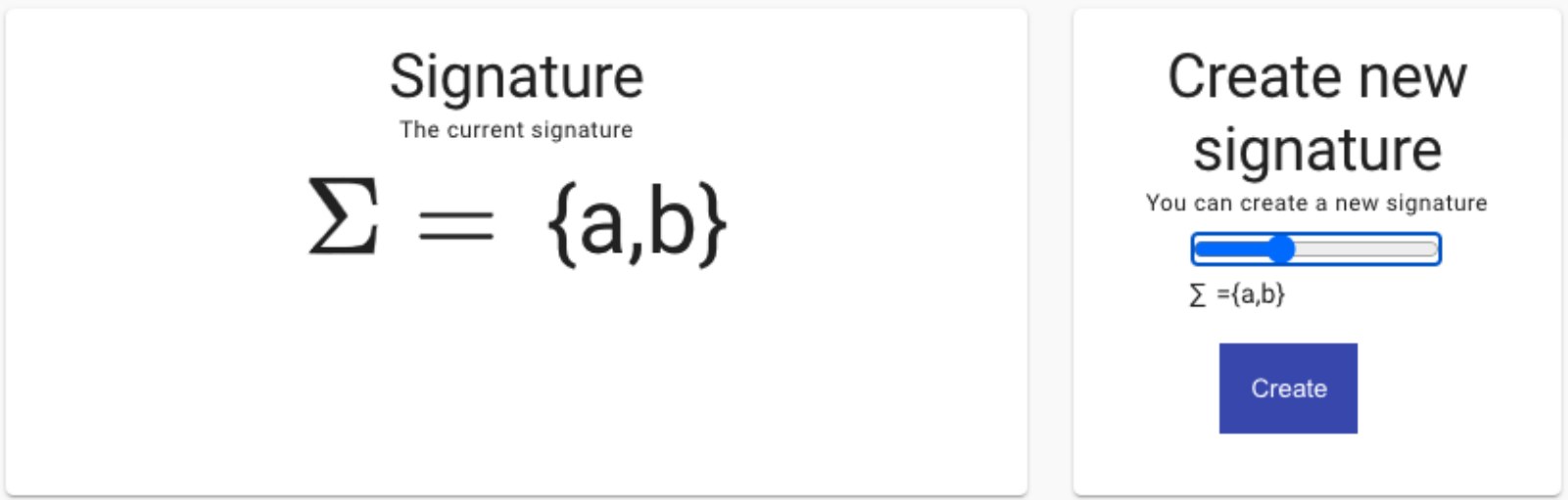
Postulate Check Server



Packages


dmf-revision
Alchourron
Preference Builder
logic-components
logic-ts

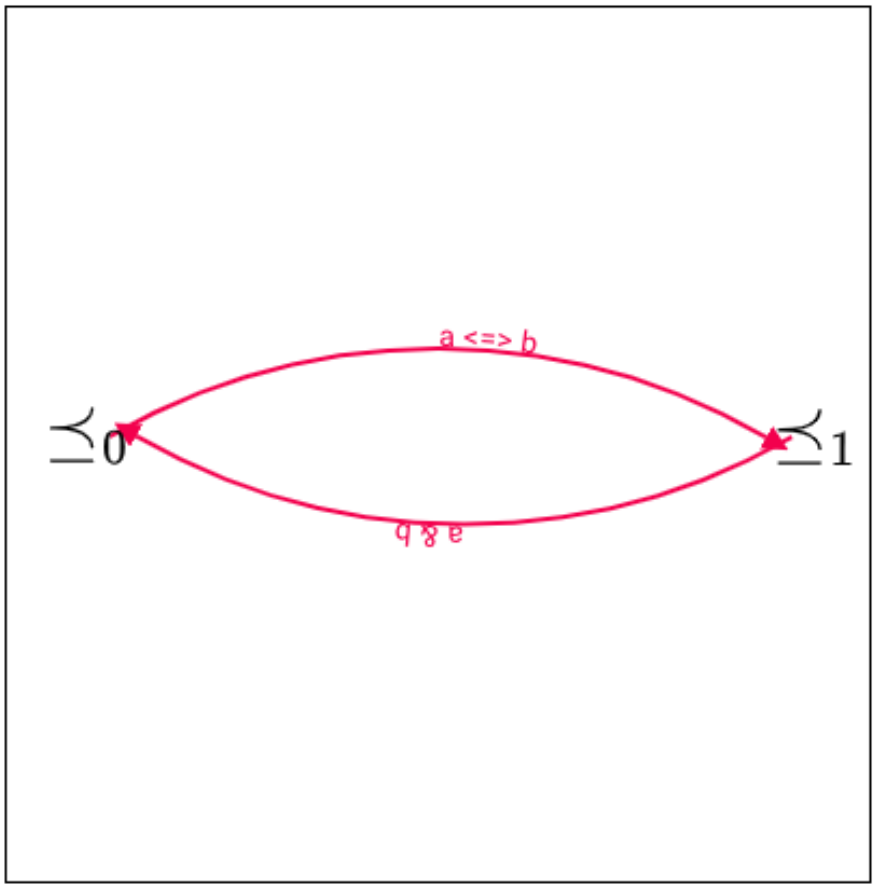
Signature


Signature
The current signature
Σ = {a,b}
Create new signature
You can create a new signature
Σ ={a,b}
Create

System Graph

EDIT AS LIST | EDIT AS GRAPH

Graph



Edit Element

$$\preceq_0 \in T$$

2	
1	$\overline{a}\overline{b}$
0	ab $\quad$ $\overline{a}b$ $\quad$ $a\overline{b}$

$$e_0\colon (\preceq_0, a \leftrightarrow b, \preceq_1)$$

from

$\preceq_0$

Formula in TPTP syntax (tptp.org)

a <=> b

to

$\preceq_1$

System List

EDIT AS LIST | EDIT AS GRAPH

Total preorders $\preceq \in T$

$\preceq_0$

$\preceq_1$

ADD

Edges $e \in E$

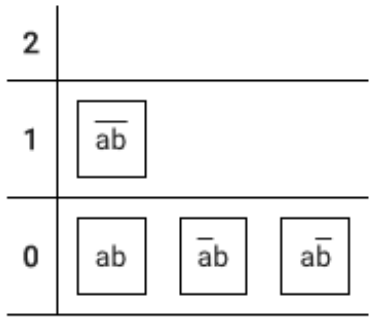
e_0: $(\preceq_0, a \leftrightarrow b, \preceq_1)$

e_1: $(\preceq_1, (a \wedge b), \preceq_0)$

ADD

Edit Element

$\preceq_0 \in T$



e_0: $(\preceq_0, a \leftrightarrow b, \preceq_1)$

from

$\preceq_0$

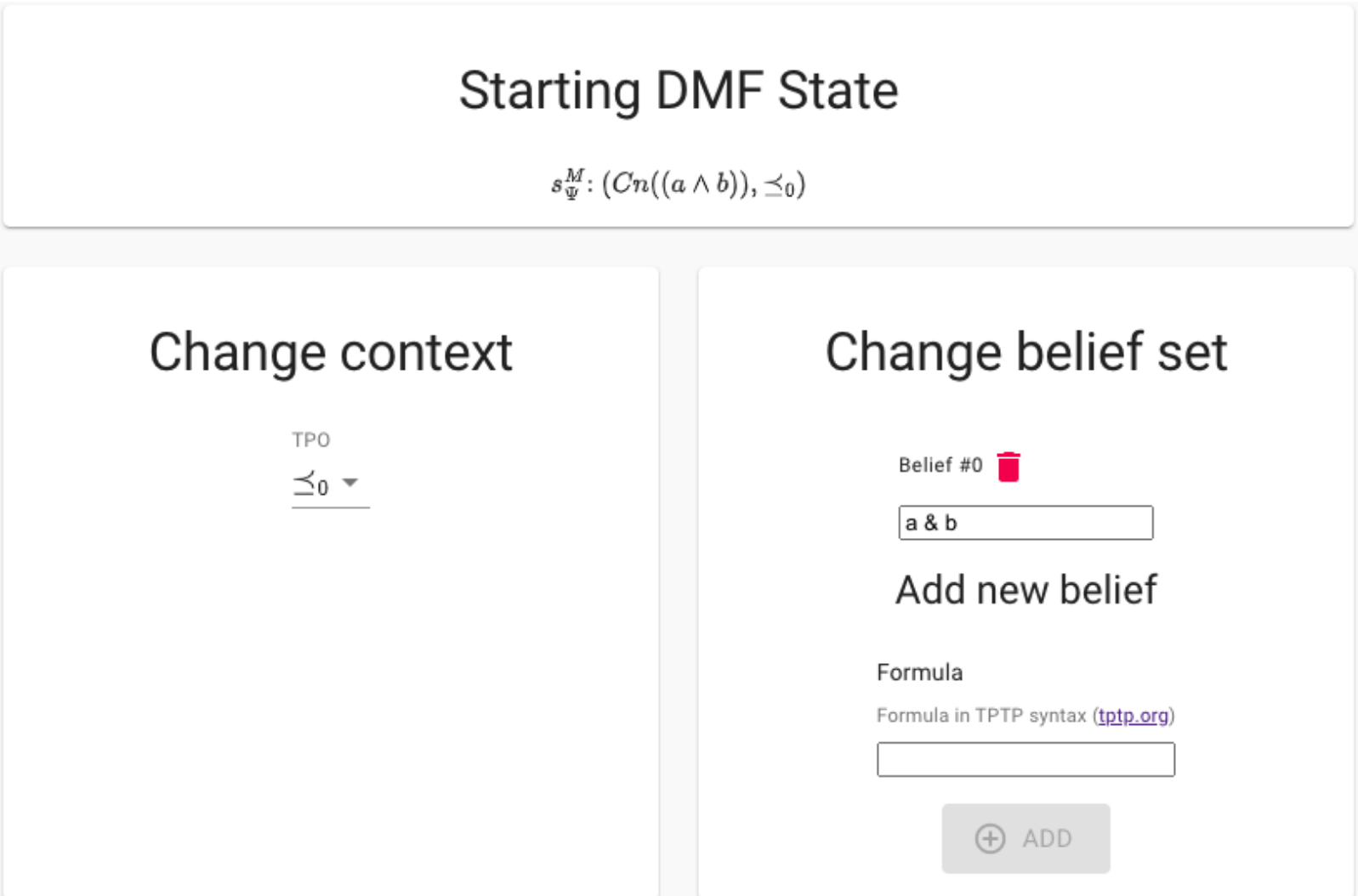
Formula in TPTP syntax (tptp.org)

a <=> b

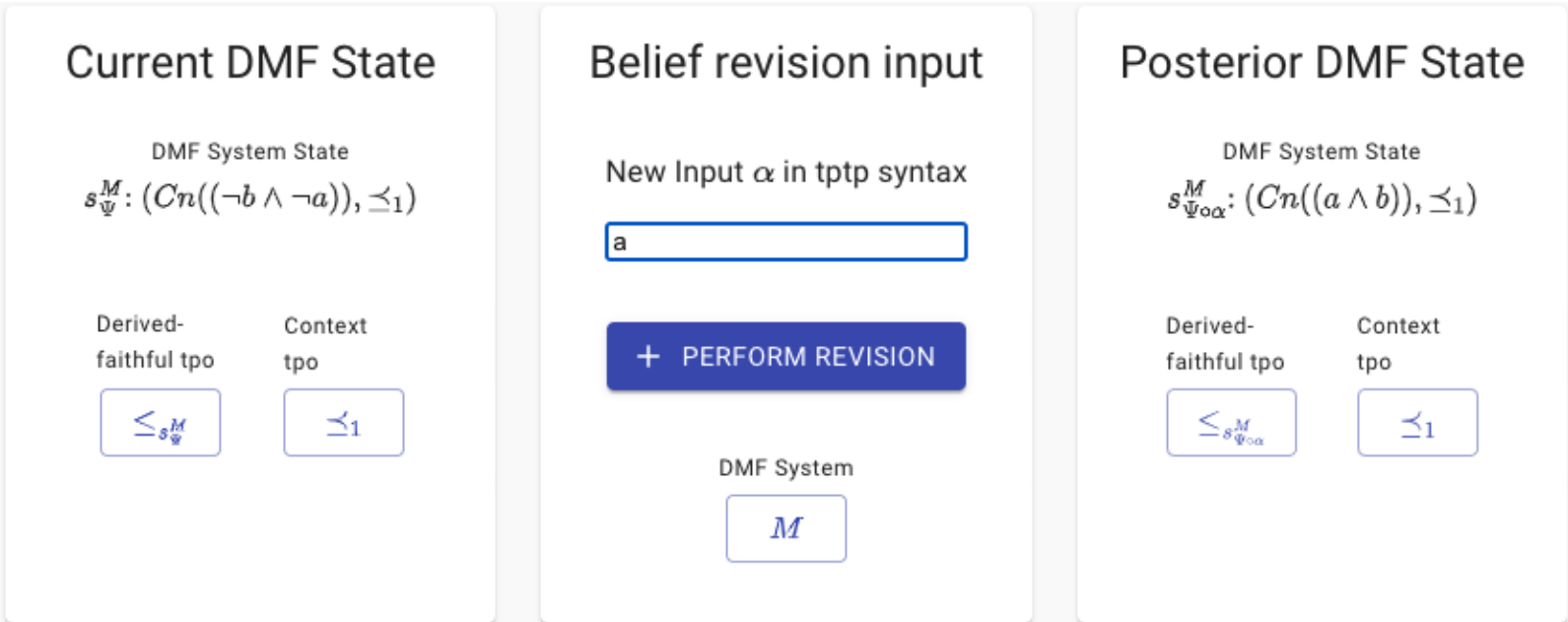
to

$\preceq_1$

DMF State


Starting DMF State
$s_{\Psi}^{M}: (Cn((a \wedge b)), \preceq_0)$
Change context
TPO
$\preceq_0$
Change belief set
Belief #0
a & b
Add new belief
Formula
Formula in TPTP syntax (tptp.org)
ADD

Revision Input



Revision Configured

Configured iterated belief revision steps

Revision #2

Prior DMF State

DMF System State

$s^{M}_{\Psi_2}\colon (Cn((a \wedge b)), \preceq_1)$

Derived-faithful tpo: $\leq_{s^{M}_{\Psi_2}}$

Context tpo: $\preceq_1$

Input:

$\neg a$

Posterior DMF State

DMF System State

$s^{M}_{\Psi_3}\colon (Cn((\neg b \wedge \neg a)), \preceq_1)$

Derived-faithful tpo: $\leq_{s^{M}_{\Psi_3}}$

Context tpo: $\preceq_1$

Revision #1

Prior DMF State

DMF System State

$s^{M}_{\Psi_1}\colon (Cn((a \wedge b)), \preceq_0)$

Derived-faithful tpo: $\leq_{s^{M}_{\Psi_1}}$

Context tpo: $\preceq_0$

Input:

$a \leftrightarrow b$

Edge transition

$e_0\colon (\preceq_0, a \leftrightarrow b, \preceq_1)$

Posterior DMF State

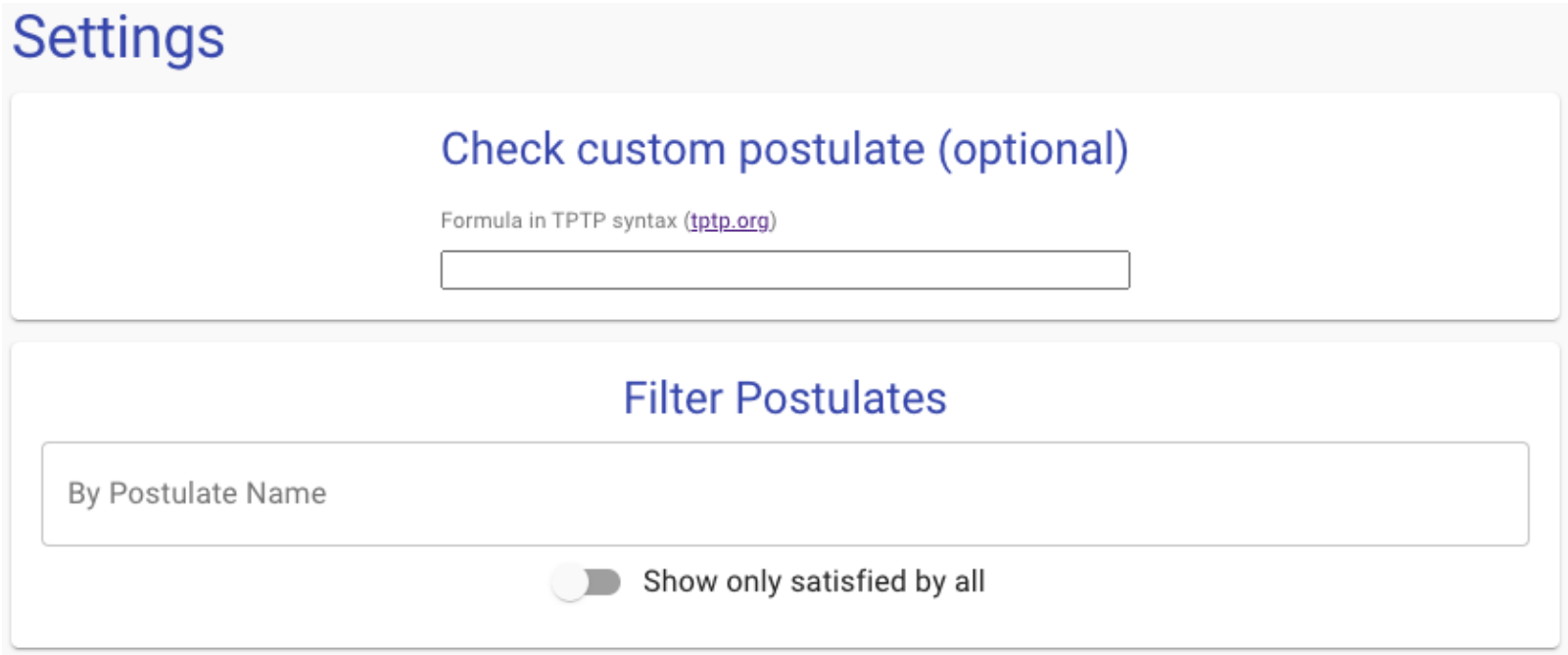
DMF System State

$s^{M}_{\Psi_2}\colon (Cn((a \wedge b)), \preceq_1)$

Derived-faithful tpo: $\leq_{s^{M}_{\Psi_2}}$

Context tpo: $\preceq_1$

Result Settings



Individual Result

Revision #1

Prior DMF State

DMF System State

$s^M_{\Psi_1}: (Cn((a \wedge b)), \preceq_0)$

Derived-faithful tpo: $\leq_{s^M_{\Psi_1}}$

Context tpo: $\preceq_0$

Input:

$a \leftrightarrow b$

Edge transition

$e_0: (\preceq_0, a \leftrightarrow b, \preceq_1)$

Posterior DMF State

DMF System State

$s^M_{\Psi_2}: (Cn((a \wedge b)), \preceq_1)$

Derived-faithful tpo: $\leq_{s^M_{\Psi_2}}$

Context tpo: $\preceq_1$

Postulate	Satisfied?	Formula
CR1	✓	SHOW
CR2	✓	SHOW
CR3	✓	SHOW
CR4	✓	SHOW
CR5	×	SHOW
CR6	×	SHOW

Result Overview

Results Overview

Postulate	Revision #1	Revision #2
CR1	✓	✓
CR2	✓	✓
CR3	✓	✓
CR4	✓	✓
CR5	×	✓
CR6	×	✓

Evaluation and Improvements

Software Quality

- choosing JSON and TPTP
- automated tests and builds
- Web components and React

Performance

- signature size has biggest impact
- postulate evaluation performance depends on amount of quantors
- some work already done [SH21]

Thank you

- Philip Heltweg, pheltweg@gmail.com
- Try yourself online at `http://coeus.rhazn.com/`

References I

[AGM85] Carlos E. Alchourrón, Peter Gärdenfors, and David Makinson. "On the logic of theory change: Partial meet contraction and revision functions". In: *Journal of Symbolic Logic* 50.2 (June 1985), pp. 510–530. DOI: 10.2307/2274239.

[Ara20] Theofanis Aravanis. "On uniform belief revision". In: *Journal of Logic and Computation* 30.7 (Sept. 2020), pp. 1357–1376. DOI: 10.1093/logcom/exaa058.

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[Has+20] Wilhelm Hasselbring et al. "From FAIR research data toward FAIR and open research software". In: *it - Information Technology* 62.1 (Feb. 2020), pp. 39–47. DOI: 10.1515/itit-2019-0040.

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[McC56] Edward J McCluskey. "Minimization of Boolean functions". In: *The Bell System Technical Journal* 35.6 (1956), pp. 1417–1444.

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[SH21] Kai Sauerwald and Philip Heltweg. "On Using Model Checking for the Certification of Iterated Belief Changes". In: *Proceedings of the 7th Workshop on Formal and Cognitive Reasoning co-located with the 44th German Conference on Artificial Intelligence (KI 2021), Berlin, Germany, September 28, 2021*. Ed. by Christoph Beierle et al. Vol. 2961. CEUR Workshop Proceedings. CEUR-WS.org, 2021, pp. 23–33. URL: http://ceur-ws.org/Vol-2961/paper_3.pdf.