Certification of Iterated Belief Changes via Model Checking and its Implementation

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Abstract

Iterated belief change investigates principles for changes on epistemic states and their representational groundings. A common realisation of epistemic states are total preorders over possible worlds. In this paper, we consider the problem of certifying whether an operator over total preorders satisfies a given postulate. We introduce the first-order fragment $\mathrm{FO}^{\mathrm{TPC}}$ for expressing belief change postulates and present a way to encode information on changes into an $\mathrm{FO}^{\mathrm{TPC}}$-structure. As a result, the question of whether a belief change fulfils a postulate becomes a model checking problem. We present *Alchourron*, an implementation of our approach, consisting of an extensive Java library, and also of a web interface, which suits didactic purposes and experimental studies.

1 Introduction

A fundamental problem for intelligent agents is adapting their world-view to potentially new and conflicting information. Iterated belief change discusses properties of operators that model transition of currently held beliefs under newly received information. The field has a large body of literature with differentiated results for a variety of different types of operations, e.g., revision (Darwiche and Pearl 1997; Booth, Meyer, and Wong 2006), contraction (Hild and Spohn 2008; Konieczny and Pino Pérez 2017; Sauerwald, Kern-Isberner, and Beierle 2020), expansion, the area of non-prioritized change (Konieczny and Pino Pérez 2008; Booth et al. 2014; Schwind and Konieczny 2020) and many more (Schwind, Konieczny, and Marquis 2018).

The research on (iterated) belief change is focussed on propositional logic (but not limited to). Often, total preorders over interpretations (Darwiche and Pearl 1997; Konieczny and Pino Pérez 2008; Booth et al. 2014; Schwind, Konieczny, and Marquis 2018; Sauerwald, Kern-Isberner, and Beierle 2020; Schwind and Konieczny 2020; Konieczny and Pino Pérez 2017; Schwind and Konieczny 2020) or refinements thereof (Hild and Spohn 2008; Booth, Meyer, and Wong 2006) are considered as a representation formalism for epistemic states.

A common aspect of many approaches in the area of iterated belief change is that the type of an operator class is given by syntactic postulates, constraining how to change, and that representation theorems show, which semantic postulates exactly reconstruct that class of operations in the realm of total preorders. The typical structure of postulates, regardless of whether there are syntactic or semantic postulates; is that they focus on a single (but arbitrary) epistemic state and constrain the result of subsequent changes on that state. When total preorders are considered as epistemic states, then very often, the so-called faithfulness condition and a representation theorem connects the syntactic viewpoint with the semantic perspective, e.g. (Darwiche and Pearl 1997).

Given the large variety of different postulates and types of operations, it is tedious and cumbersome to check manually whether a given specific change satisfies a certain postulate, or to decide whether the change falls into a certain category of type of operation.

This leads to the general problem of checking whether a belief change operator or a singular change satisfies a given syntactic or semantic postulate, which we call the *certification problem*. The certification problem got not much attention, notable exceptions are results about the complexity for specific operations (Nebel 1998; Liberatore 1997; Schwind et al. 2020) and results about inexpressibility (Turán and Yaggie 2015). Furthermore, there seems to be no implementation for the certification problem for the area of iterated belief change.

In this paper, we propose an approach to grasp the certification problem for the case where total preorders are used as epistemic states and provide an implementation. The approach consists of defining the first-order fragment $\mathrm{FO}^{\mathrm{TPC}}$, which is meant as a language for semantic postulates. To focus on semantic postulates seems to be only a minor restriction, as, given the many representation theorems, many syntactic postulates are known to be expressible by semantic postulates in the total preorder realm. Second, we describe how an $\mathrm{FO}^{\mathrm{TPC}}$-structure can be constructed for a belief change operator and for a singular belief change, respectively. The certification problem then becomes a first-order model-checking problem. Third, we present an implementation of the approach, which is publically available on the web.

2 Belief Change on Epistemic States

Let $\mathcal{L}$ be a propositional language over a finite signature of propositional variables Σ, and Ω its corresponding set of interpretations. Following the framework of Darwiche and Pearl (Darwiche and Pearl 1997), we deal with belief changes over epistemic states and propositions. An epistemic

Predicate	Intended meaning	Exemplary appearance
$Mod(w,x)$	w is a model of x	$\omega \in \mathrm{Mod}(\Psi), \omega \in \mathrm{Mod}(\alpha)$
$LessEQ(w_1,w_2,e)$	$w_1 \leqslant w_2$ in e	$\omega_1 \leqslant_\Psi \omega_2$
$Int(w)$	w is an interpretation	$\omega \in \Omega$
$ES(e)$	e is an epistemic state	$\Psi \in \mathcal{E}$
$Form(a)$	a is a formula	$\alpha \in \mathcal{L}$
Function	**Intended meaning**	**Exemplary appearance**
$op(e_0,a)$	$op(e_0,a)$ is a result of changing e_0 by a	$\Psi * \alpha = \Psi'$
$or(a,b)$	propositional disjunction	$\mathrm{Bel}(\Psi \circ (\alpha \vee \beta)) = \dots$
$not(a)$	propositional negation	$\neg\alpha \notin \mathrm{Bel}(\Psi \circ \alpha)$

Figure 1: Allowed predicates and functions symbols in $\mathrm{FO}^{\mathrm{TPC}}$, their intended meaning and how they are typically formulated in belief change literature.

state is an abstract entity from a set $\mathcal{E}$, where each $\Psi \in \mathcal{E}$ is equipped with a deductively closed set $\mathrm{Bel}(\Psi)$. A belief change operator is a function $\circ : \mathcal{E} \times \mathcal{L} \to \mathcal{E}$. In this paper, we only consider operators satisfying the following syntax-independence condition for each $\Psi \in \mathcal{E}$ and $\alpha, \beta \in \mathcal{L}$:

(sAGM5es*) if $\alpha \equiv \beta$, then $\Psi \circ \alpha = \Psi \circ \beta$

Here (sAGM5es*) is a stronger version of the *extensionality* postulate from the revision approach by Alchourrón, Gärdenfors and Makinson (1985) (AGM).

The framework by Darwiche and Pearl is different from the classical setup for belief revision theory by Alchourrón, Gärdenfors and Makinson (1985), where deductively closed sets (called belief sets) are used as states (Fermé and Hansson 2011). However, the richer structure of epistemic states is necessary to include the information required to capture the change strategy of iterative belief change (Darwiche and Pearl 1997).

There are many possible instantiations of $\mathcal{E}$; however, we will stick here to the very common one by total preorders. More precisely, we consider total preorders over Ω that fulfil the so-called faithfulness condition (Katsuno and Mendelzon 1992; Darwiche and Pearl 1997), stating that the minimal elements of each total preorder $\leqslant = \Psi \in \mathcal{E}$ are exactly the models of $\mathrm{Bel}(\Psi)$, i.e., $\mathrm{Mod}(\mathrm{Bel}(\Psi)) = \min(\Omega, \leqslant)$. Thus, in the scope of this paper, each total preorder $\leqslant \in \mathcal{E}$ is assumed to entirely describe an epistemic state.

3 Problem Statement

Postulates are central objects in the area of (iterative) belief change and are grouped together to define classes of belief change operators in a descriptive way. The problem we address is to check whether a given operator satisfies a postulate, i.e., belongs to a class of change operators specified by postulates. We call this particular problem the certification problem (which could be considered as a generalisation of the revision problem (Nebel 1998)):

CERTIFICATION-PROBLEM
Given: A belief change operator $\circ$ and a postulate P
Question: Does $\circ$ satisfy the postulate P?

Clearly, information about a whole belief change operator is available or even finitely representable only in few application scenarios. This gives rise to several sub-problems depending on how much information of the particular operator is known. Apart from the full operator $\circ$, we consider the certification of the following cases:

- A singular belief change from Ψ to Ψ' by α, i.e.: Does $\Psi \circ \alpha = \Psi'$ hold?
- A sequence of belief changes $\Psi_1 \circ \alpha_1 = \Psi_2$, and $\Psi_2 \circ \alpha_2 = \Psi_3$, and ...
- All singular belief changes on a state Ψ, i.e. the set $\{(\Psi_1, \alpha, \Psi_2) \in \circ \mid \Psi = \Psi_1\}$

In the next section we present a model-checking based formalisation of the CERTIFICATION-PROBLEM.

4 The Approach

In belief change, postulates are usually described by common mathematical language, which is close to (first-order) predicate logic. In the following, we use the toolset of first-order logic to formalise the CERTIFICATION-PROBLEM as a first-order model-checking problem.

Language for Postulates As an initial study, we considered several postulates from literature on iterated belief change, e.g. (Darwiche and Pearl 1997; Booth and Meyer 2006; Jin and Thielscher 2007; Booth 2002; Nayak, Pagnucco, and Peppas 2003), and selected the most common predicates and functions used. We compiled them into a fragment of first-order logic with equality over a fixed set of predicates and function symbols[1], denoted by $\mathrm{FO}^{\mathrm{TPC}}$ (Total Preorder Change), with the intention to describe changes over total preorders. Figure 1 summarises the permitted symbols and describes only the minimal required set.

Several common predicates and functions used in postulates are expressible by the means of $\mathrm{FO}^{\mathrm{TPC}}$ by employing this minimal set, e.g. logical entailment, semantic equality, the strict part of a total preorder, checking whether a formula has no model, etc. For a specific example, consider the following:

$$LogImpl(x,y) := \forall w. Int(w) \to (Mod(w,x) \to Mod(w,y))$$

[1]Note that we could also use a fragment of many-sorted first-order logic. However, some predicates are "overloaded" in respect to sorts.

Universe	$U^{\mathcal{A}_C} = \Omega \cup \{\Psi_0, \Psi_1\} \cup \mathcal{P}(\Omega)$			
Predicates				
$Mod^{\mathcal{A}_C}$	$= \{(\omega, x) \mid x \in \mathcal{P}(\Omega) \cup \{\Psi_0, \Psi_1\}, \omega \in \mathrm{Mod}(x)\}\}$			
$Int^{\mathcal{A}_C}$	$= \Omega$			
$ES^{\mathcal{A}_C}$	$= \{\Psi_0, \Psi_1\}$			
$Form^{\mathcal{A}_C}$	$= \mathcal{P}(\Omega)$			
$LessEQ^{\mathcal{A}_C}$	$= \{(\omega_1, \omega_2, \Psi_i) \mid \omega_1 \leqslant_{\Psi_i} \omega_2\}$			
Functions				
$or^{\mathcal{A}_C}$	$= \lambda\alpha_1, \alpha_2.\, \alpha_1 \cup \alpha_2$	$e_0^{\mathcal{A}_C}$	$= \Psi_0$	
$not^{\mathcal{A}_C}$	$= \lambda\alpha_1.\, \Omega \setminus \alpha_1$	$a^{\mathcal{A}_C}$	$= \mathrm{Mod}(\alpha)$	
$op^{\mathcal{A}_C}$	$= (\{(\Psi, \beta, \Psi) \mid \beta \in \mathcal{P}(\Omega), \Psi \in \{\Psi_0, \Psi_1\}\} \setminus \{(\Psi_0, \alpha, \Psi_0)\}) \cup \{(\Psi_0, \alpha, \Psi_1)\}$			

Figure 2: Structure $\mathcal{A}_C$, encoding a singular change $C = (\Psi_0, \alpha, \Psi_1)$

where $LogImpl(x, y)$ describes that x logically implies y.

For illustration, we consider some aspects about belief change postulates. First, belief change postulates are typically formulated with a locality aspect; every postulate focusses an initial state and a change formula α, describing a condition for this change. As prominent examples, the following postulates are an excerpt of the AGM revision postulates (Alchourrón, Gärdenfors, and Makinson 1985):

(AGM2*) $\alpha \in \mathrm{Bel}(\Psi \circ \alpha)$

(AGM7*) $\mathrm{Bel}(\Psi \circ (\alpha \wedge \beta)) \subseteq Cn(\mathrm{Bel}(\Psi \circ \alpha) \cup \{\beta\})$

In $\mathrm{FO}^{\mathrm{TPC}}$, we address this by reserving e_0 and a as special terms, where e_0 denotes the initial state and a denotes the formula representing the new information.

Postulates for (iterated) belief change typically come in two fashions: *Semantic postulates* describe changes in a semantic domain, such as faithful total preorders. For example, consider the following postulate:

(CR1) if $\omega_1, \omega_2 \in \mathrm{Mod}(\alpha)$, then $\omega_1 \leqslant_\Psi \omega_2 \Leftrightarrow \omega_1 \leqslant_{\Psi\circ\alpha} \omega_2$

This could be expressed in $\mathrm{FO}^{\mathrm{TPC}}$ by the following formula $\varphi_{\text{(CR1)}}$:

$$\begin{aligned}\varphi_{\text{(CR1)}} = &\forall w1, w2.\\ &(Int(w_1) \wedge Int(w_2) \wedge ES(e_0) \wedge Form(a))\\ &\rightarrow (LessEQ(w_1, w_2, e_0)\\ &\quad \leftrightarrow LessEQ(w_1, w_2, op(e_0, a)))\end{aligned} \tag{1}$$

On the other hand, *syntactic postulates* describe changes of $\mathrm{Bel}(\Psi)$. Aside of the AGM revision postulates, prominent examples are the Darwiche-Pearl postulates for revision (Darwiche and Pearl 1997) such as:

(DP1) if $\beta \models \alpha$, then $\mathrm{Bel}(\Psi \circ \alpha \circ \beta) = \mathrm{Bel}(\Psi \circ \beta)$

Several representation results in the literature show how syntactic and semantic postulates are interrelated. For instance, it is well-known that, given $\circ$ is an AGM revision operator, (CR1) holds if and only if (DP1) holds (Darwiche and Pearl 1997). Moreover, the semantic and syntactic domains are of course related, which allows us to describe many predicates used in the syntactic realm by semantic means. For example, a statement like $\mathrm{Bel}(\Psi \circ \alpha \circ \beta) = \mathrm{Bel}(\Psi \circ \beta)$ is expressible in $\mathrm{FO}^{\mathrm{TPC}}$ by employing the following formula:

$$\begin{aligned}Bel(a, e) := &(Form(a) \wedge ES(e))\\ &\rightarrow (\forall x. Mod(x, a) \leftrightarrow Mod(x, e))\end{aligned}$$

We describe now how objects like belief change operators, singular changes and so on are related to $\mathrm{FO}^{\mathrm{TPC}}$ formulas.

Encoding as Model-Checking Internally, we use the standard truth-functional semantics of first-order logic for $\mathrm{FO}^{\mathrm{TPC}}$. Therefore, we translate a belief change operator, respectively the known part of it, into a first-order structure.

The general idea is to define a structure $\mathcal{A}$ by the following pattern: The universe $U^{\mathcal{A}}$ consists of all propositional interpretations Ω, all formulas from $\mathcal{L}$ and all considered epistemic states from Ψ, i.e., the total preorders over Ω. We represent formulas by their models, i.e., by elements of[2] $\mathcal{P}(\Omega)$. The rationale is that, because of (sAGM5es*), the considered belief change operators are insensitive to syntactic differences. Additionally, predicates are interpreted in the straight-forward manner, e.g., Int is interpreted as all propositional interpretations, $Int^{\mathcal{A}} = \Omega$, and $LessEQ$ allows access to the total preorder Ψ of each epistemic state, $LessEQ^{\mathcal{A}} = \{(\omega_1, \omega_2, \Psi_i) \mid (\omega_1, \omega_2) \in \Psi_i\}$. Depending on whether a full change operator, a singular change, or another sub-problem is considered, some special treatment is necessary.

For instance, consider the signature $\Sigma = \{a, b\}$, yielding the interpretations $\Omega = \{ab, \overline{a}b, a\overline{b}, \overline{a}\overline{b}\}$. Moreover, consider the singular change $C = (\Psi_0, \alpha, \Psi_1)$, where $\Psi_0 = \leqslant_0$ is the total preorder treating every interpretation to be equally plausible, i.e., $ab =_0 a\overline{b} =_0 \overline{a}b =_0 \overline{a}\overline{b}$. Furthermore, let $\alpha = a$. The total preorder $\Psi_1 = \leqslant_1$ treats all a-models to be equally plausible, but prefers them over all non a-models, which are considered to be equally plausible, i.e. $ab =_1 a\overline{b} <_1 \overline{a}b =_1 \overline{a}\overline{b}$. We construct a structure $\mathcal{A}_C$ as follows: The universe is given by $U^{\mathcal{A}_C} = \Omega \cup \{\Psi_0, \Psi_1\} \cup \mathcal{P}(\Omega)$. The predicates and function symbols are interpreted according to Figure 2. The terms e_0 and a are interpreted as $e_0^{\mathcal{A}_C} = \Psi_0$ and $a^{\mathcal{A}_C} = \mathrm{Mod}(\alpha)$.

[2] $\mathcal{P}(\cdot)$ is the powerset function.

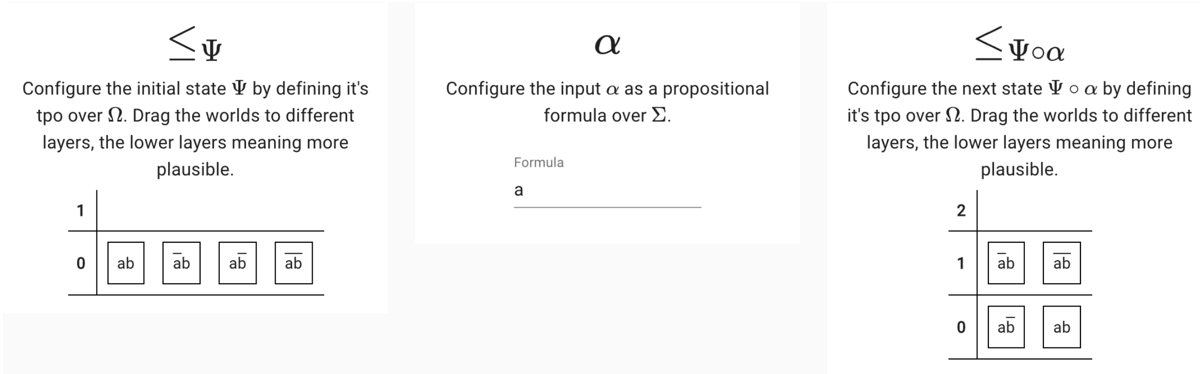

Figure 3: Input fields for the change from Ψ_0 to Ψ_1 by $\alpha = a$ in Alchourron.

In summary, the CERTIFICATION-PROBLEM of whether C satisfies (CR1) is expressed as a model-checking problem for $\mathrm{FO}^{\mathrm{TPC}}$, i.e., a change C satisfies the postulate (CR1) if $\mathcal{A}_C \models \varphi_{(\mathrm{CR1})}$ holds, where $\varphi_{(\mathrm{CR1})}$ is the formula given in (1).

5 Implementation

We provide an implementation of the approach by combining independent, self-developed Java libraries. The approach is publicly accessible by a web-frontend called *Alchourron*[3], which expands on the previous work by Sauerwald and Haldimann (Sauerwald and Haldimann 2019). The currently available version allows the specification of a singular belief change using a browser-based client. First, the user decides on a propositional signature for the language of the belief change. Then a prior total preorder, an input formula, as well as the posterior total preorder is entered. Figure 3 illustrates the belief change input.

After specifying the change, Alchourron allows the user to check whether several preconfigured belief change postulates are satisfied. Optionally, a user can also enter her own postulate by defining a first-order formula using $\mathrm{FO}^{\mathrm{TPC}}$. Formulas are described in TPTP syntax (Sutcliffe 2017), e.g., the postulate (CR1) from Section 4 can be expressed as follows:

```
! [W1,W2] :
 ((int(W1) & int(W2) & mod(W1, A) & mod(W2,
    A))
=> (lesseq(W1, W2, E0)
   <=> lesseq(W1, W2, op(E0, A))))
```

Internally, Alchourron has a client-server architecture. The implementation is highly modularized, and we expect reusability of components for further projects. In particular, postulate checking via compilation into a model-checking problem as described in Section 4 is happening completely on the server side. Display of total preorders is provided by web components[4] that can also represent *ordinal conditional functions* (Spohn 1988), which for instance implement total preorders, but provide also more fine-grained representations of epistemic states. Our implementation of logic is an extensive institution-inspired implementation called *Logical Systems*[5], which allows representation and evaluation of a variety of different logics in a unified way. Preconfigured postulates are stored in TPTP syntax and parsed from there[6], mapping TPTP specified formula into our internal representation.

6 Summary and Future Work

We proposed $\mathrm{FO}^{\mathrm{TPC}}$, a first-order fragment to describe belief change postulates, complemented with a methodology to construct a finite structure for a belief change operator, employing total preorders as representation of epistemic states. With this toolset, the certification of belief change operators can be understood as a model-checking problem. We presented our implementation, which is available online[3], as a proof of concept for our approach for singular belief changes. In summary, we defined and formalized the certification problem and provide an implementation therefore.

While this is only the first proposal, we expect that this approach will be highly flexible regarding improvements and extensions. In particular, for future work we want to expand our approach to more complex representations of epistemic states. Moreover, we will work to improve the efficiency of the implementation.

[3]Visit: https://www.fernuni-hagen.de/wbs/alchourron/

[4]Heltweg, P.: Logic components, 2021. DOI: 10.5281/zenodo.4744650.

[5]Sauerwald, K.: Logical Systems, 2021, github.com/Landarzar/logical-systems.

[6]Steen, A.: Scala TPTP parser, 2021. DOI: 10.5281/zenodo.4672395.

References

Alchourrón, C. E.; Gärdenfors, P.; and Makinson, D. 1985. On the logic of theory change: Partial meet contraction and revision functions. *J. Symb. Log.* 50(2):510–530.

Booth, R., and Meyer, T. A. 2006. Admissible and restrained revision. *J. Artif. Intell. Res.* 26:127–151.

Booth, R.; Fermé, E. L.; Konieczny, S.; and Pino Pérez, R. 2014. Credibility-limited improvement operators. In Schaub, T.; Friedrich, G.; and O'Sullivan, B., eds., *ECAI 2014 - 21st European Conference on Artificial Intelligence, 18-22 August 2014, Prague, Czech Republic*, volume 263 of *Frontiers in Artificial Intelligence and Applications*, 123–128. IOS Press.

Booth, R.; Meyer, T. A.; and Wong, K. 2006. A bad day surfing is better than a good day working: How to revise a total preorder. In Doherty, P.; Mylopoulos, J.; and Welty, C. A., eds., *Proceedings, Tenth International Conference on Principles of Knowledge Representation and Reasoning, Lake District of the United Kingdom, June 2-5, 2006*, 230–238. AAAI Press.

Booth, R. 2002. On the logic of iterated non-prioritised revision. In Kern-Isberner, G.; Rödder, W.; and Kulmann, F., eds., *Conditionals, Information, and Inference, International Workshop, WCII 2002, Hagen, Germany, May 13-15, 2002, Revised Selected Papers*, volume 3301 of *Lecture Notes in Computer Science*, 86–107. Springer.

Darwiche, A., and Pearl, J. 1997. On the logic of iterated belief revision. *Artificial Intelligence* 89:1–29.

Fermé, E. L., and Hansson, S. O. 2011. AGM 25 years - twenty-five years of research in belief change. *J. Philosophical Logic* 40(2):295–331.

Hild, M., and Spohn, W. 2008. The measurement of ranks and the laws of iterated contraction. *Artif. Intell.* 172(10):1195–1218.

Jin, Y., and Thielscher, M. 2007. Iterated belief revision, revised. *Artif. Intell.* 171(1):1–18.

Katsuno, H., and Mendelzon, A. O. 1992. Propositional knowledge base revision and minimal change. *Artif. Intell.* 52(3):263–294.

Konieczny, S., and Pino Pérez, R. 2008. Improvement operators. In Brewka, G., and Lang, J., eds., *Principles of Knowledge Representation and Reasoning: Proceedings of the Eleventh International Conference, KR 2008, Sydney, Australia, September 16-19, 2008*, 177–187. AAAI Press.

Konieczny, S., and Pino Pérez, R. 2017. On iterated contraction: syntactic characterization, representation theorem and limitations of the Levi identity. In *Scalable Uncertainty Management - 11th International Conference, SUM 2017, Granada, Spain, October 4-6, 2017, Proceedings*, volume 10564 of *Lecture Notes in Artificial Intelligence*. Springer.

Liberatore, P. 1997. The complexity of iterated belief revision. In Afrati, F. N., and Kolaitis, P. G., eds., *Database Theory - ICDT '97, 6th International Conference, Delphi, Greece, January 8-10, 1997, Proceedings*, volume 1186 of *Lecture Notes in Computer Science*, 276–290. Springer.

Nayak, A. C.; Pagnucco, M.; and Peppas, P. 2003. Dynamic belief revision operators. *Artif. Intell.* 146(2):193–228.

Nebel, B. 1998. *How Hard is it to Revise a Belief Base?* Dordrecht: Springer Netherlands. 77–145.

Sauerwald, K., and Haldimann, J. 2019. WHIWAP: checking iterative belief changes. In Beierle, C.; Ragni, M.; Stolzenburg, F.; and Thimm, M., eds., *Proceedings of the 8th Workshop on Dynamics of Knowledge and Belief (DKB-2019) and the 7th Workshop KI & Kognition (KIK-2019), Kassel, Germany, September 23, 2019*, volume 2445 of *CEUR Workshop Proceedings*, 14–23. CEUR-WS.org.

Sauerwald, K.; Kern-Isberner, G.; and Beierle, C. 2020. A conditional perspective for iterated belief contraction. In Giacomo, G. D.; Catalá, A.; Dilkina, B.; Milano, M.; Barro, S.; Bugarín, A.; and Lang, J., eds., *ECAI 2020 - 24nd European Conference on Artificial Intelligence, August 29th - September 8th, 2020, Santiago de Compostela, Spain*, 889–896. IOS Press.

Schwind, N., and Konieczny, S. 2020. Non-Prioritized Iterated Revision: Improvement via Incremental Belief Merging. In *Proceedings of the 17th International Conference on Principles of Knowledge Representation and Reasoning*, 738–747.

Schwind, N.; Konieczny, S.; Lagniez, J.; and Marquis, P. 2020. On computational aspects of iterated belief change. In Bessiere, C., ed., *Proceedings of the Twenty-Ninth International Joint Conference on Artificial Intelligence, IJCAI 2020*, 1770–1776.

Schwind, N.; Konieczny, S.; and Marquis, P. 2018. On belief promotion. In Thielscher, M.; Toni, F.; and Wolter, F., eds., *Principles of Knowledge Representation and Reasoning: Proceedings of the Sixteenth International Conference, KR 2018, Tempe, Arizona, 30 October - 2 November 2018.*, 297–307. AAAI Press.

Spohn, W. 1988. Ordinal conditional functions: a dynamic theory of epistemic states. In Harper, W., and Skyrms, B., eds., *Causation in Decision, Belief Change, and Statistics, II*. Kluwer Academic Publishers. 105–134.

Sutcliffe, G. 2017. The TPTP Problem Library and Associated Infrastructure. From CNF to TH0, TPTP v6.4.0. *Journal of Automated Reasoning* 59(4):483–502.

Turán, G., and Yaggie, J. 2015. Characterizability in belief revision. In *Proceedings of the Twenty-Fourth International Joint Conference on Artificial Intelligence, IJCAI 2015, Buenos Aires, Argentina, July 25-31, 2015*, 3236–3242.